**MATHEMATICS SPECIALIST**

**MAWA Year 12 Examination 2019**

**Calculator-assumed**

# Marking Key

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The release date for this exam and marking scheme is 14th June.

**Question 9(a) (3 marks)**

|  |
| --- |
| Solution |
| If Now has modulus  and argument Thus the two roots of the quadratic are cis with   |
| Mathematical behaviours | Marks |
| * applies the quadratic formula appropriately
* identifies the modulus and argument for one of the roots
* deduces the second root
 | 111 |

**Question 9(b) (3 marks)**

|  |
| --- |
| Solution |
| Since  then    If  then  For this to hold, the argument must be a multiple of which implies that is a multiple of 3  |
| Mathematical behaviours | Marks |
| * states the correct form for
* derives the correct equation for the two expressions to be equal
* deduces the acceptable values of
 | 111 |

**Question 10(a) (3 marks)**

|  |
| --- |
| Solution |
| If **rij** then  and Hence  and  . Since ,  |
| Mathematical behaviours | Marks |
| * correctly determines equations for $\frac{x}{3}=cos⁡(2t)$ and $\frac{y}{4}=sin⁡(2t)$
* uses Pythagorean theorem correctly to obtain the result required
 | 21 |

**Question 10(b) (1 mark)**

|  |
| --- |
| Solution |
| The path is an ellipse |
| Mathematical behaviours | Marks |
| * states the path is an ellipse
 | 1 |

**Question 10(c) (1 mark)**

|  |
| --- |
| Solution |
|  If **rij** then **vij** |
| Mathematical behaviours | Marks |
| * derives the correct expression for the velocity vector
 | 1 |

**Question 10(d) (3 marks)**

|  |
| --- |
| Solution |
| Speed =|**v**(t)|This equals    |
|  | Marks |
| * writes down the correct expression for the speed
* rewrites
* obtains an expression for the speed in terms of
 | 111 |

**Question 10(e) (2 marks)**

|  |
| --- |
| Solution |
| Maximum speed occurs when  and then speed is If for integer Hence maximum speed attained when for integer values of  |
|  | Marks |
| * deduces the maximum speed
* states the times when maximum speed is attained
 | 11 |

**Question 11 (6 marks)**

|  |
| --- |
| Solution |
|  |
| Mathematical behaviours | Marks |
| * displays correct shape of $y=1/f(x)$ graph
* indicates correct horizontal ‘limits’ of $1/f(x)$ graph
* displays ‘reflection’ property of $y=f^{-1}(x)$ graph in
* displays ‘intersection’ points near $(2,2)$ and $(6,6)$
 | 11+111+1 |

**Question 12 (8 marks)**

|  |
| --- |
| Solution |
|  As OC=CQ and angles ; hence so co-ordinates of Q are (3,3)Also anglesand since the triangle OPC is equilateral. Hence x co-ordinate of P is so that co-ordinates of P are Area includes the boundary  |
| Mathematical behaviours | Marks |
| * draws the circle with correct centre and correct radius
* draws lines of appropriate slopes through the origin
* shades (or somehow indicates) the required area
* derives the co-ordinates of Q
* derives the co-ordinates of P
* makes statement or shows that the boundary is included in region
 | 1+11111+11 |

**Question 13 (a) (7 marks)**

|  |
| --- |
| Solution |
| Initial velocity **v****i** + **j** =( **i**+ **j**)As **a****j** then **v****j j**+ **c**Applying the initial condition yields **c**=( **i**+ **j**) so **v****i** **j**Integrating again then **r****v****i** **j** + **d** for some constant **d**Since initially **r=0** so **d**=**0,** and hence**r****i** **j** |
| Mathematical behaviours | Marks |
| * correctly obtains **v**(0)
* uses **a****j** to derive **v**
* uses initial conditions to determine the constant vector **c**
* deduces that **v****i** **j**
* anti-differentiates **v** to give **r**
* uses **r****0** to determine **d**
* deduces the final form of **r**
 | 1111111 |

**Question 13 (b) (2 marks)**

|  |
| --- |
| Solution |
| Maximum height is achieved when the vertical component of velocity vanishesThis occurs when At this time height of projectile is  metres |
| Mathematical behaviours | Marks |
| * determines time for maximum height
* correctly evaluates for maximum height of 500 m
 | 11 |

**Question 13 (c) (2 marks)**

|  |
| --- |
| Solution |
| Projectile reaches horizontal again when the **j** component of **r** vanishes.The requisite time is seconds |
|  | Marks |
| * recognition of the need to determine when the vertical displacement is zero
* correct calculation of total flight time
 | 11 |

**Question 13 (d) (2 marks)**

|  |
| --- |
| Solution |
| When  then **v**(**i** - **j**) and speed = m/s |
|  | Marks |
| * evaluation of the velocity vector when the projectile strikes ground
* evaluates the requisite speed
 | 11 |

**Question 14 (a) (7 marks)**

|  |
| --- |
| Solution |
| Since , .But this quadratic has no real roots so has no real zero.Now .Hence at and .Since and , then and are critical points.The line is a vertical asymptote for the graph.As  then as  |
|  |
| Mathematical behaviours | Marks |
| * shows that there is no zero
* differentiates correctly
* solves $f^{'}\left(0\right)=0$
* evaluates $f(1)$ and $f(3)$
* obtains vertical asymptote
* derives the correct form of the expression for large
* deduces correct limiting behaviours as $x\rightarrow \infty $ and $x\rightarrow -\infty $
 | 1111111+1 |

**Question 14 (b) (5 marks)**

|  |
| --- |
| Solution |
|  |
| Mathematical behaviours | Marks |
| * shows vertical asymptote correctly
* shows critical points correctly
* indicates where the curve cuts the y-axis
* displays oblique asymptote correctly
 | 11+111 |

**Question 15(a) (3 marks)**

|  |
| --- |
| Solution |
| We know that  so that  Then   so that  as required.  |
| Mathematical behaviours | Marks |
| * writes down appropriate form for
* uses appropriate properties to write this exponential in terms of and
* deduces the requisite result
 | 111 |

**Question 15(b) (6 marks)**

|  |
| --- |
| Solution |
| If we denote Raising to the appropriate power gives   Hence so that as required  |
| Mathematical behaviours | Marks |
| * raises the result of part (a) to the fifth power
* expands the binomial correctly (both coefficients and signs)
* collects powers together in the appropriate way
* writes in terms of
* deduces the required answer
 | 11+1111 |

**Question 16(a) (6 marks)**

|  |
| --- |
| Solution |
| If  is a factor of then this means that Hence ………….(\*)Equating imaginary parts tells us that Now this gives that either  or The real part of (\*) gives that Now if this equation cannot hold so this possibility must be ruled out.Then if Hence we conclude that and   |
| Mathematical behaviours | Marks |
| * appreciates that is a factor implies that
* expands the form of
* puts real and imaginary parts of the expression both equal to zero
* shows that imaginary part yields three possible values for
* argues that the form of the real part prohibits the possibility
* hence deduces the appropriate value of
 | 111111 |

**Question 16(b) (4 marks)**

|  |
| --- |
| Solution |
| Now we deduce that is a factor of By long division or CAS If Hence the four roots of are  and   |
| Mathematical behaviours | Marks |
| * identifies the quadratic factor of
* determines the other quadratic factor
* solves for the other two roots of the equation
* states all four solutions in explicit form
 | 1111 |

**Question 17 (a) (3 marks)**

|  |
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| Solution |
|  |
| Mathematical behaviours | Marks |
| * plots the three graphs reasonably accurately

 (blue curve is , purple and red ) | 1+1+1 |

**Question 17(b) (4 marks)**

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| --- |
| Solution |
| $$f^{'}\left(x\right)=A+\cos(x)$$So $A-1\leq f^{'}\left(x\right)\leq A+1.$Now $f$ is $1-1\leftrightarrow f'$ does no change sign. (\*) So $f$ is $1-1\leftrightarrow A\geq 1$ or $A\leq -1$ |
| Mathematical behaviours | Marks |
| * differentiates correctly
* notes the range of values of the derivative
* identifies criterion for $1-1$ property (\*)
* answers correctly
 | 1111 |

**Question 17(c) (3 marks)**

|  |
| --- |
| Solution |
| To evaluate $f^{-1}(5)$ we need to solve $f\left(x\right)=-2x+\sin(x)=5$ (\*)From a calculator $x≅-2.71$ |
| Mathematical behaviours | Marks |
| * writes down equation (\*)
* gives solution to the specified level of accuracy
 | 12 |

**Question 18 (a) (6 marks)**

|  |
| --- |
| Solution |
| Let A denote the airplane. Given the position and velocity in the question, at time  after 1pm the airplane is located at Similarly, at the same time the helicopter is located atThe first components of the two position vectors coincide when When the aircraft is located at  The helicopter is located at  |
| Mathematical behaviours | Marks |
| * correctly determines the position of the airplane at time
* correctly determines the position of the helicopter at time
* determines when one the components of the two locations are equal
* shows that at this time the other two components are also equal
* concludes with evidence that a collision is imminent
 | 1121 1 |

**Question 18 (b) (2 marks)**

|  |
| --- |
| Solution |
| Since  the collision occurs at 3.36pm and at the location **i****j****k**  |
| Mathematical behaviours | Marks |
| * correctly converts $t=2.6$ to 3.36 p.m.
* states position of the collision point
 | 1 1 |

**Question 18 (c) (7 marks)**

|  |
| --- |
| Solution |
| At 2 p.m., $t=1$ so$r\_{A}\left(1\right)= <60, 160, 3.74>+1<-100, 20, 0.8> = <-40, 180, 4.54>km$ and$$r\_{H}\left(1\right)= <-70, 108, 4.52>+1<-50, 40, 0.5> = <-120, 148, 5.02>km$$Making the helicopter to be at rest (by imposing a negative velocity on it) we have:$$ \_{A}v\_{H}=v\_{A}-v\_{H}=<-150, 120, 0.5> - <-50, 40, 0.5> = <-100, 80, 0>$$$$\vec{HA}=\vec{HO}+\vec{OA}= -<-120, 148, 5.02>+ <-40, 180, 4.54> = <80, 32, -0.48>$$$\vec{HR}=\vec{HA}+ \_{A}v\_{H}$ (where *R* is the point at which airplane and helicopter are closest) $=<80, 32, -0.48>+ t<-100, 80, 0> $$$ = <80-100t, 32+80t, -0.48>$$Calculate $\vec{HR} ∙$ $ \_{A}v\_{H}=0$ to determine closest distance between aircrafti.e. $<80-100t, 32+80t, -0.48> ∙ <-100, 80, 0> =0$Simplifying gives: $5440=16400t \rightarrow t=0.3317$ hoursAt $t=0.3317$ , $\left|\vec{HR}\right|=\left|<46.83, 58.536, -0.48>\right|=74.96 $km$∴$ the shortest distance between the aircraft following the redirection is 74.96 km |
| Mathematical behaviours | Marks |
| * determines the position vectors $r\_{A}\left(1\right)$ and $r\_{H}\left(1\right)$
* determines relative velocity vector for $ \_{A}v\_{H}$
* correctly develops equation (i.e. $\vec{HR}=\vec{HA}+ \_{A}v\_{H}$)
* uses scalar dot product $\vec{HR} ∙$ $ \_{A}v\_{H}=0$
* evaluates for $t=0.7508$ hours
* determines minimum distance following the redirection
 | 211111 |